

Non-commutative geometry inspired higher-dimensional charged, black holes[★]

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Abstract

We obtain a new, exact, solution of the Einstein's equation in higher dimensions. The source is given by a static spherically symmetric, Gaussian distribution of mass and charge. The resulting metric describes a regular, i.e. curvature singularity free, charged black hole in higher dimensions. The metric smoothly interpolates between Reissner-Nordström geometry at large distance, and deSitter spacetime at short distance. Thermodynamical properties of the black hole are investigated and the form of the Area Law is determined. We study pair creation and show that the upper bound on the discharge time increases with the number of extra dimensions.

[★] As this paper was nearing completion our friend Gallieno Denardo has suddenly passed away. As his former students, we dedicate this paper to his memory and strongly believe that without his early influence it probably would have never been written.

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1 Introduction

By now, it can be safely claimed that string theory induced non-commutative geometry [1] provides an effective framework to study short distance space-time dynamics. The original idea, revived by the today stringy formulation, dates back to Snyder seminal paper [2]. In this model there exist a universal minimal length scale. At distances near $\sqrt{\theta}$ the classical concept of smooth spacetime manifold breaks down. It is generally assumed that $\sqrt{\theta}$ is closed to the Planck length, and as such it would be inaccessible both to present and future experimental observations.

A promising alternative to the Planck scenario has been proposed, recently, based on the presence of “large extra-dimensions”. It allows to lower unification scale, i.e. string tension, down to TeV energy so that stringy effects could be soon observed at LHC [3]. Thus, if non commutative geometry is induced by string theory, the corresponding length scale should be lowered as well. These ideas offer exciting, near future, possibility of experimentally probing both non-commutativity and quantum gravity effects. The most convincing confirmation of TeV *Quantum Gravity* would be the production of a mini black hole (BH) in hadronic collisions [4],[5],[6][7]. An unambiguous signature of such an event is based on a detailed analysis of both decay products and of the eventual BH remnant. So far the standard scenario of BH evaporation offered a detailed and definite predictions for the spectrum of emitted particles, while it is inconclusive about the final phase of BH evaporation. This ambiguity can be traced back to the semi-classical description of Hawking process in the sense that matter emitted by BH is represented by quantum field theory in curved spacetime, while BH itself is still a *classical* background geometry. On the other hand, the final stage of BH decay requires quantum gravity corrections which the semi-classical model is unable to provide.

In view of the “quantum” character of non-commutative geometry, one expects that in this model the late stage of the evaporation is determined by spacetime fluctuations when the radius of the BH horizon becomes comparable with $\sqrt{\theta}$. Usual attempts to obtain BH solutions of “non-commutative” Einstein’s equations are based upon perturbative expansion in the $\sqrt{\theta}$ parameter. So far, all these attempts lead to unconvincing results: it is unacceptable that curvature singularities can survive coordinate fluctuations in spite of the existence of a minimal length [8]. It is likely that these problems are due to the method of approximating the original, intrinsically non-local theory, with a local truncated expansion of the action functional in terms of derivative interactions.

Against this background, we proposed an approach based on “quasi-classical coordinates”, keeping track of the intrinsic non-locality in the form a minimal length in the spacetime fabric. As a result, matter/energy densities are described by minimal width Gaussian distributions, in complete agreement with the basic principles of quantum mechanics. Solutions of Einstein’s equations

with such smeared sources give new kind of regular BHs in four [10] and higher dimensions [11]. These models lead to new predictions for the final stage of Hawking evaporation. End result is a massive remnant in the form of a neutral, cold, near-extremal BH. The mass of this residual and stable object is determined by the θ parameter. Its eventual observation would be an unambiguous experimental signature pointing out that:

- i) a BH has been produced in the hadronic collision;
- ii) there exists a minimal length inherent to short distance spacetime structure [14][15].

Motivated by this exciting perspective, we present in this paper a detailed study of the higher dimensional, charged, regular, mini BH.

2 Higher dimensional solution

Before we engage in a detailed calculation we would like to point out the basic idea of our implementation of non-commutative effects. It would be desirable to formulate non-commutative theories, including General Relativity, directly in terms of non-commuting coordinates. This approach would correspond to a new, deeper, level of “quantization” acting on the spacetime manifold itself rather than on fields, including the metric, which are structures assigned over the manifold itself. Unfortunately a framework of this type is not presently available.

In a non-commutative geometry, familiar concepts lose their very meaning. As an example, it is worth to remark a problem that seems to be ignored in the current literature [8] while it should be especially evident when facing non-commutative extensions of General Relativity. Coordinate non-commutativity implies the existence of a *finite* minimal length $\sqrt{\theta}$, below which the very concept of “*distance*” becomes physically meaningless. Such a basic remark, immediately, raises the problem to define the line element, namely the *infinitesimal* distance between two nearby points in Einstein gravity. One possible way around is to adopt Weyl-Wigner-Moyal \star product [16], paying the cost of possible violation of unitarity [17] and Lorentz invariance in ordinary quantum field theory, or anisotropy of the Newton gravitational potential ⁴[18]. Furthermore, star-product formulation of non-commutative quantum field theory can be handled only through perturbative expansion in the θ parameter. At any finite order in θ the approximated theory keeps no memory of its original non-locality and looks like an ordinary quantum field theory affected by higher order derivative self-interactions. The original UV cut-off $1/\sqrt{\theta}$ is turned into a dimensional coupling constant leading to a non-normalizable UV behavior

⁴ The alternative approach in terms of coordinate coherent states leads to a spherical symmetry preserving, short distance modification only of the Newton potential [19]

even worse than in ordinary quantum field theory.

For all these reasons, we have developed an effective approach where non-commutativity is implemented only through a Gaussian de-localization of matter sources. In this way there is no problem in defining line element and Einstein's equations are kept unchanged. Our strategy can be summarized as follows: i) in non-commutative geometry there cannot be point-like object, because any physical distance cannot be smaller than a minimal position uncertainty of the order of $\sqrt{\theta}$; ii) this effect is implemented in spacetime through matter de-localization, which by explicit calculations [20] turns out to be of Gaussian form; iii) Spacetime geometry is determined through Einstein's equations with de-localized matter sources; iv) de-localization of matter backfires at spacetime geometry giving regular, i.e. curvature singularity free, metrics. This is exactly what is expected from the existence of a minimal length.

The presence of a universal short distance cut-off leads to the following effects: in quantum field theory it cures UV divergences [21]; in General Relativity it cures curvature singularities [10,22].

In view of the above explanations, we are going to solve the resulting Einstein's equations, in higher dimensions, with a maximally localized source of energy and electric charge. The corresponding distributions have a minimal spread Gaussian profile

$$\rho_{\mu_0}(r) = \frac{\mu_0}{(4\pi\theta)^{m/2}} \exp(-r^2/4\theta) , \quad (1)$$

$$\rho_q(r) = \frac{q}{(4\pi\theta)^{m/2}} \exp(-r^2/4\theta) \quad (2)$$

Matter and charge density in Eq.(1), (2) model a physical source which is as close as it is possible to a “point-like” object. μ_0 is the “bare mass” and q is the total electric charge. We remark that μ_0 is only part of the total mass-energy of the system. The Coulomb energy stored in the electric field is a second contribution to the total mass-energy sourcing the gravitational field.

We are looking for static, spherically symmetric gravitational and electric fields solving the coupled field equations

$$R^\mu{}_\nu - \frac{1}{2} \delta^\mu{}_\nu R = 8\pi G (T^\mu{}_\nu|_{\text{matt.}} + T^\mu{}_\nu|_{\text{el.}}) \quad (3)$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\mu\nu}) = J^\nu , \quad J^\mu(x) = 4\pi \rho_q(r) \delta_0^\mu \quad (4)$$

$$F^{\mu\nu} = \delta^{0[\mu} \delta^{r|\nu]} E(r) \quad (5)$$

where the Greek indices μ, ν , etc. run over $0, 1, 2, \dots, m$; $G = M_*^{1-m}$ is the reduced fundamental scale for $m \geq 4$ i.e. $M_* \sim 1/\sqrt{\theta} \approx 1 \text{ TeV}$, while for

$m = 3$, $G = M_{Pl}^{-2}$ i.e. $M_{Pl} \sim 1/\sqrt{\theta} \approx 1.22 \times 10^{16} \text{ TeV}$.

$T_\nu^\mu|_{\text{matt.}}$ is the higher dimensional extension of the energy-momentum tensor described in [10]; J^μ is charge current which has 0-component non-vanishing. The Coulomb-like field satisfying the equation (5) results to be

$$E(r) = \frac{2\Gamma(m/2)}{\pi^{(m-2)/2}} \frac{q_\theta(r)}{r^{m-1}}, \quad q_\theta(r) = \frac{q}{\Gamma(m/2)} \gamma\left(\frac{m}{2}, \frac{r^2}{4\theta}\right) \quad (6)$$

where, $\gamma(a/b; x)$ is the Euler lower Gamma function defined by the following integral representation:

$$\gamma(a/b, x) \equiv \int_0^x \frac{du}{u} u^{a/b} e^{-u} \quad (7)$$

It turns out that the electric field is linearly vanishing for $r \rightarrow 0$ independently of the number of space dimensions. On the other hand, the large distance behavior is sensitive to extra dimensions and falls-off as $1/r^{m-1}$.

In order to proceed, we insert solution (6) in the electromagnetic energy momentum tensor and solve Einstein's equations (3). We find Reissner-Nordström like form of the metric :

$$ds_{(m+1)}^2 = -h(r) dt^2 + h(r)^{-1} dr^2 + r^2 d\Omega_{m-1}^2$$

$$h(r) = 1 - \frac{4G\mu_0}{\pi^{(m-2)/2} r^{m-2}} \gamma\left(\frac{m}{2}, \frac{r^2}{4\theta}\right) + (m-2) \frac{4q^2 G}{\pi^{m-3} r^{2m-4}} F(r) \quad (8)$$

$$F(r) \equiv \gamma^2\left(\frac{m}{2} - 1, \frac{r^2}{4\theta}\right) - \frac{2^{(8-3m)/2} r^{m-2}}{(m-2)\theta^{(m-2)/2}} \gamma\left(\frac{m}{2} - 1, \frac{r^2}{2\theta}\right) \quad (9)$$

The main feature of (8) is the absence of curvature singularities. To prove this statement in a simple and painless way, we consider the short-distance behavior of the metric. Inserting the small argument expansion of (7) in (8), the geometry near the origin is given by

$$h(r) = 1 - \frac{4G\mu_0}{m 2^{m-1} \pi^{(m-2)/2} \theta^{m/2}} r^2 + O(r^4) \quad (10)$$

$$\Lambda_\theta \equiv \frac{4G\mu_0}{m 2^{m-1} \pi^{(m-2)/2} \theta^{m/2}} \quad (11)$$

One recognizes in (10) deSitter spacetime, which is known to be singularity-free. Equation (11) shows as the bare mass μ_0 and the noncommutative parameter θ mix together to produce an *effective cosmological constant* Λ_θ . It is not surprising that different mass particles experience different cosmological constants. The deSitter vacuum represents the geometrical counterpart of the un-

derlying noncommutative coordinate fluctuations which are taking place over a distance scale defined by $\sqrt{\theta}$. Different values of μ_0 imply different Compton wavelength and “resolution power”. Heavy particles can probe shorter distance better than light objects, and result to be more sensible to spacetime quantum fluctuations. In our solution the curvature singularity smears into a regular, fluctuating, vacuum core which is quasi-classically described by a deSitter geometry. Short-distance spacetime fuzziness, once it is implemented through our quasi-classical geometrical method, cures curvature singularity as it was anticipated in the Introduction.

In order to proceed in our investigation, it is convenient to define the *radial* mass-energy, $\mu(r)$ accounting for *both* Coulomb and matter energy contributions

$$\mu(r) \equiv \frac{2\mu_0}{\pi^{\frac{m-2}{2}}} \gamma\left(\frac{m}{2}, \frac{r^2}{4\theta}\right) + \frac{2^{\frac{8-3m}{2}}}{\pi^{m-3}} \frac{2q^2}{\theta^{\frac{m}{2}-1}} \gamma\left(\frac{m}{2} - 1, \frac{r^2}{2\theta}\right) \quad (12)$$

The total mass-energy, M , measured by an asymptotically distant observer is given by

$$M = \lim_{r \rightarrow \infty} \mu(r) = \frac{2\mu_0}{\pi^{\frac{m-2}{2}}} \Gamma\left(\frac{m}{2}\right) + \frac{2^{\frac{8-3m}{2}}}{\pi^{m-3}} \frac{2q^2}{\theta^{\frac{m}{2}-2}} \Gamma\left(\frac{m}{2} - 1\right) \quad (13)$$

Solution 8) is now expressed in terms of M as

$$h(r) = 1 - \frac{2MG}{r^{m-2} \Gamma\left(\frac{m}{2}\right)} \gamma\left(\frac{m}{2}, \frac{r^2}{4\theta}\right) + (m-2) \frac{4Gq^2}{\pi^{m-3} r^{2m-4}} \left[F(r) + d_m r^{m-2} \gamma\left(\frac{m}{2}, \frac{r^2}{4\theta}\right) \right] \quad (14)$$

$$d_m \equiv \frac{2^{\frac{8-3m}{2}}}{m-2} \frac{1}{\theta^{\frac{m-2}{2}}} \frac{\Gamma\left(\frac{m}{2} - 1\right)}{\Gamma\left(\frac{m}{2}\right)} \quad (15)$$

As one might expect, at distance $r \gg \sqrt{\theta}$, Equation(8) gives ordinary m -dimensional Reissner-Nordström metric

$$h(r) \longrightarrow 1 - \frac{2MG}{r^{m-2}} + (m-2) \frac{4Gq^2}{\pi^{m-3} r^{2m-4}} \Gamma^2\left(\frac{m}{2} - 1\right) \quad (16)$$

At this point, it is important to address the problem of the existence of event horizons. Their position is determined by the roots of the equation $h(r_+) = 0$. In our case horizon equation cannot be solved analytically, but a plot of $h(r)$

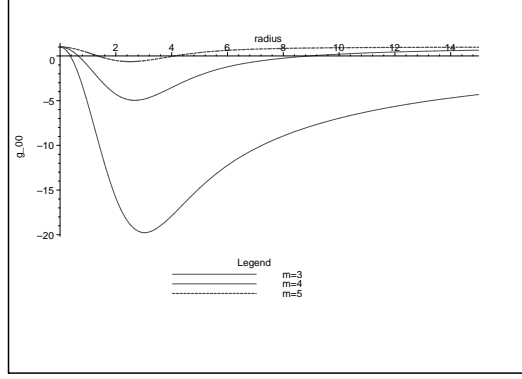


Fig. 1. $h(r)$ versus $r/\sqrt{\theta}$, for a charge $Q \equiv q M_*^{(m-1)/2} = \theta^{(m-2)/2}$ and $M = 40 M_*$. We can observe that the inner radius decreases with m , while the outer horizon radius increases with m .

as a function of r , for fixed values of M , q , m determines the positions of horizon(s).

For any m there can be two horizons when $M > M_0$, one degenerate horizon for $M = M_0$, or no horizon if $M < M_0$. M_0 is the mass of an *extremal* BH and represents its final state at the end of Hawking evaporation process. The details will be given in next Section. Furthermore, it results that increasing m , more and more mass is needed to create a black hole of a given radius. The global properties of the analytically extended solution across the horizon (coordinate singularities) can be obtained by gluing together outer Reissner-Nordström and inner deSitter patches. On general ground horizons are solutions of the equation

$$M = U(r_H; q) \quad (17)$$

$$U(r_H; q) = 2^{\frac{8m-3}{2}} \Gamma\left(\frac{m-2}{2}\right) \frac{2q^2}{\pi^{m-3}} \frac{1}{\theta^{\frac{m-2}{2}}} + \frac{\Gamma\left(\frac{m}{2}\right)}{2G\gamma(m/2, r_{\pm}^2/4\theta)} \left[r_H^{m-2} + \frac{(m-2)4Gq^2}{\pi^{m-3} r_H^{m-2}} F(r_H) \right] \quad (18)$$

Eq.(18) offers an alternative way of studying existence of horizons.

The use of the equation relating the total mass energy of the system to the radius of the event horizon follows the approach proposed in [23] with the advantage of allowing an in-depth investigation of geometry and dynamics of the system. We see that the sole effect of extra-dimensions is to lift upward the $3+1$ dimensional curve thus increasing the value of the mass for a given radius of the horizon, including the degenerate case. Thus, we conclude that the evolution of the BH towards its extremal configuration is qualitatively the same as described in [10]. The increase of the mass M_0 and its eventual experimental verification could indicate the number of extra-dimensions.

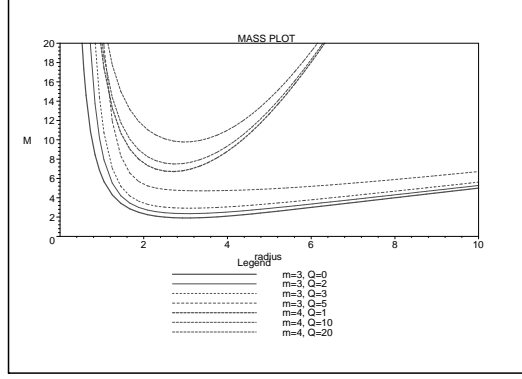


Fig. 2. $U(r_H ; q)$ for different q and m .

3 Black hole thermodynamics

In this section we are going to investigate some thermodynamic properties of the regular BH described by the line element (8). The starting point is the explicit form of the Hawking temperature T_H :

$$4\pi T_H = \frac{1}{r_+} \left[m - 2 - r_+ \frac{\gamma'(m/2, r_+^2/4\theta)}{\gamma(m/2, r_+^2/4\theta)} \right] +$$

$$- \frac{16 G q^2}{\pi^{m-3} r_+^{2m-3}} \left[\gamma^2(m/2, r_+^2/4\theta) + (m-2) \frac{r_+}{4} F(r_+) \frac{\gamma'(m/2, r_+^2/4\theta)}{\gamma(m/2, r_+^2/4\theta)} \right] \quad (19)$$

where, we replaced M with r_+ by using the horizon equation (18). From the

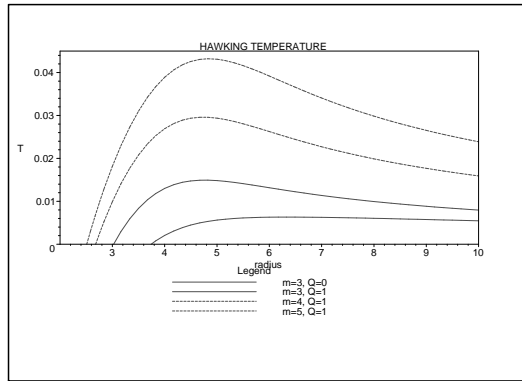


Fig. 3. Hawking temperature $T_H \sqrt{\theta}$ as a function of $r_+/\sqrt{\theta}$, for different values of $Q/\sqrt{\theta}^{(m-2)}$ and m . T_H reaches a maximum whose value increases with m before dropping to zero.

plot of the temperature one sees that in any dimension there is an initial “Schwarzschild-phase” for $r_+ \geq 7\sqrt{\theta}$. Approaching maximum temperature,

a departure from the Schwarzschild behavior shows up. The maximum temperature occurs in the range $4.5\sqrt{\theta} \leq r_+ \leq 7\sqrt{\theta}$. Immediately after the temperature drops down and the BH enters a *nearly-extremal* phase, asymptotically approaching a zero-temperature, degenerate configuration. This is the same behavior already encountered in the neutral case [10]. The effect of extra-dimensions is to shift upwards the temperature while shrinking the radius of the extremal BH. The effect of charge is just to lower the temperature as can be seen from the two curves with $m = 3$ and $Q = 0, 1$. Therefore, upper bounds for the maximal temperature can be obtained from the plot of T_H in the neutral case.

One can see from (19) that the peak temperature increases with m , but even in the case $m = 10$ it is about 98 GeV ($\simeq 10^{15} \text{ K}$) which is much lower than M_* . For $m = 3$ i.e. no extra-dimensions, the fundamental scale $M_* = M_{Pl.}$. Notwithstanding, back-reaction is still negligible as the maximal temperature is $18 \times 10^{16} \text{ GeV}$ much lower than $T_{Pl.}$.

Table 1

BH remnant (minimal) masses and (minimal) radii for different values of m					
m	3	5	6	9	10
M_0 (TeV)	2.3×10^{16}	24	94	7.3×10^3	3.4×10^4
r_0 (10^{-4} fm)	4.88×10^{-16}	4.95	4.75	4.46	4.40

The energy of the electrostatic field increases the total mass-energy. Thus, the minimal value for M_0 as a function of m can be obtained studying the neutral case $Q = 0$. This is given in Table (1), where we also listed the corresponding radius of the event horizon. The BH production cross section is simply the area of the event horizon, which leads to an estimate of the order of 10 nb , for every m . This is “good news”. On the other hand, the “bad news” is that for $m \geq 6$ the remnant is too heavy to be produced at LHC [11]. Maybe, these kind of objects could be detected in Ultra-High-Energy cosmic rays [12,13].

In this section we are also going to investigate eventual effect of the minimal length on the expression of the BH entropy [24] in terms of the area of the event horizon.

It can be useful to recall that the celebrated area/entropy law was originally derived by Bekenstein in the framework of information theory. The correct proportionality constant between entropy and area of the event horizon was later obtained by Hawking through a thermodynamic approach. It can be worth to remark that this relation holds for classical ⁵, black, point-like, singular, objects, and there is no *a priori* evidence that it would keep the

⁵ Quantum effects introduce logarithmic corrections.

same form in our case. We must derive it “from scratch”, so to say. Under this respect, the thermodynamic approach is easier to carry out. We start from the fundamental law of black hole thermodynamics $dM = T_H dS_H + \phi_H dq$ where, $\phi_H \equiv \phi(r_+)$ is the electrostatic potential on the event horizon. As physical quantities are evaluated on the event horizon, M can be expressed in terms of $U(r_+; q)$ and dM can be written as

$$dM = \frac{\partial U}{\partial r_+} dr_+ + \frac{\partial U}{\partial q} dq \quad (20)$$

From the two different forms of dM one finds the following expression for the BH entropy

$$dS_H = \frac{1}{T_H} \frac{\partial U}{\partial r_+} dr_+ \quad (21)$$

We start from (18) and calculate $\partial U / \partial r_+$

$$\begin{aligned} \frac{\partial U}{\partial r_+} = & \frac{\Gamma(m/2) r_+^{m-3}}{2G\gamma(m/2, r_+^2/4\theta)} \left\{ m - 2 - r_+ \frac{\gamma'(m/2, r_+^2/4\theta)}{\gamma(m/2, r_+^2/4\theta)} + \right. \\ & \left. - \frac{16Gq^2}{\pi^{m-3} r_+^{2m-4}} \left[\gamma^2(m/2, r_+^2/4\theta) + (m-2) \frac{r_+}{4} F(r_+) \frac{\gamma'(m/2, r_+^2/4\theta)}{\gamma(m/2, r_+^2/4\theta)} \right] \right\} \end{aligned}$$

$\partial U / \partial r_+$ is confronted with the expression (19) for the BH temperature leading to the relation

$$\frac{\partial U}{\partial r_+} = 2\pi r_+^{m-2} \frac{\Gamma(m/2)}{\gamma(m/2, r_+^2/4\theta)} T_H \quad (22)$$

The final expression for the BH entropy variation is

$$dS_H = 2\pi r_+^{m-2} \frac{\Gamma(m/2)}{\gamma(m/2, r_+^2/4\theta)} dr_+ \quad (23)$$

In order to obtain the BH entropy we integrate (23) from the extremal horizon r_e up to a generic external horizon r_+ . The result is

$$\begin{aligned} \Delta S_H = & \frac{2\pi}{m-1} \Gamma(m/2) \left[\frac{r_+^{m-1}}{\gamma(m/2, r_+^2/4\theta)} - \frac{r_e^{m-1}}{\gamma(m/2, r_e^2/4\theta)} \right] + \\ & + \frac{2\pi}{m-1} \Gamma(m/2) \int_{r_e}^{r_+} dx x^{m-1} \frac{\gamma'(m/2, x^2/4\theta)}{\gamma(m/2, x^2/4\theta)^2} \end{aligned} \quad (24)$$

Taking into account that the area of the event horizon is $A_H = \frac{2\pi^{m/2}}{\Gamma(m/2)} r_+^{m-1}$ we rewrite (24) as

$$\Delta S_H = \frac{\Gamma(m/2)}{\pi^{\frac{m}{2}-1} (m-1)} \frac{1}{G_\theta(r)} (A_+ - A_e) + \delta S_H \quad (25)$$

$$G_\theta(r_+) = \frac{G}{\Gamma\left(\frac{m}{2}\right)} \gamma\left(\frac{m}{2}, \frac{r_+^2}{4\theta}\right) \quad (26)$$

The first term in equation (25) generalizes the celebrated four dimensional relation $S_H = A_H/4G_N$ to higher dimensions. It is worth noticing that the Newton constant is replaced by the effective gravitational coupling. A similar conclusion has been recently attained, in a different framework, in [25].

Finally, let us analyze in more details correction term δS_H in (25). It can be seen that $r_e > \sqrt{\theta}$ always, so we can approximate $\gamma(m/2, r_e^2/4\theta) \approx \Gamma(m/2)$ and write (in Planck units $G = 1$)

$$\delta S_H \approx \frac{1}{2^{m-2}} \frac{1}{\theta^{\frac{m}{2}-1}} \left(r_e^{2m-3} e^{-r_e^2/4\theta} - r_+^{2m-3} e^{-r_+^2/4\theta} \right) \quad (27)$$

We conclude that the area law is maintained up to exponentially small corrections.

We have shown that the temperature of a Reissner-Nordström type BH never diverges. Thus, the end-point of the Hawking evaporation is a nearly extremal BH in thermal equilibrium with the environment. However, in the four dimensional picture this is not a satisfactory candidate to the role of remnant because charged BHs results to be quantum mechanically unstable under pair production. A mini BH decays into a neutral Schwarzschild type object much before reaching a significant temperature. Once in the Schwarzschild phase the temperature will increase without limit leading to an unpredictable final stage. This is not a problem in our case thanks to the presence of the θ parameter even in neutral phase.

4 Pair creation

In this section we are going to study the discharge process with a special attention to the way the presence of extra dimension affects the mean life of the charged object.

The creation of e^\pm pairs near the event horizon is described by the Schwinger formula [26]. This formula implies that in order for creation process to take place, the electric field has to exceed the *critical intensity* $E_c \equiv m_e^2/e$. Over-criticality condition leads to

$$\frac{2q}{\pi^{\frac{m-2}{2}} r^{m-1}} \gamma \left(\frac{m}{2}, \frac{r^2}{4\theta} \right) \geq \frac{m_e^2}{e} \quad (28)$$

The concept of Dyadosphere has been introduced in [27,28] as the spherical region, of radius r_{ds} , where (28) is valid. Recently, there has been a criticism about the possibility of Dyadosphere development for astrophysical objects [29]. Without going into this debate, we would like to notice that in the case of micro BH the condition for the existence of the Dyadosphere is always met in any dimension. In our case, the radius r_{ds} is determined by

$$r_{ds}^{m-1} = \frac{2eq}{\pi^{(m-2)/2} m_e^2} \gamma \left(\frac{m}{2}, \frac{r_{ds}^2}{4\theta} \right) \quad (29)$$

Equation (29) can be solved numerically. However, in what follows we do not need the explicit value of r_{ds} . In order to proceed, we introduce a surface charge density as:

$$\sigma(r) = \frac{q_\theta(r)}{A} = q_\theta(r) \frac{\Gamma\left(\frac{m}{2}\right)}{2\pi^{m/2} r^{m-1}} \quad (30)$$

The idea is to divide the Dyadosphere into “thin” spherical shells of thickness equal to the electron Compton wavelength λ_e . Within each shell the electric field can be considered constant and described by $E(r) = 4\pi\sigma(r)$. Such a description in terms of constant field is necessary in order to apply the Schwinger formula.

The *critical surface density* is obtained when $E = E_c$. It can be inferred from Fig.(1) and the definition (29) that $r_{ds} > r_+ \geq r_0 > \sqrt{\theta}$. Thus, we see that the dependence of the critical density from m is confined only within the electric charge. For $\sigma \geq \sigma_c$ e_\pm -pairs are created and the rate of their production is given by

$$W = \frac{1}{2^{m+1}\pi^m} [4\pi e\sigma]^{(m+1)/2} \exp\left(-\frac{m_e^2}{4e\sigma}\right) \quad (31)$$

The total number of pairs produced, in one second, inside a spherical shell of thickness $\lambda_e = 1/2m_e$ is

$$\frac{\Delta N}{\Delta\tau} \equiv \lambda_e A(r) W = \frac{\lambda_e r^{m-1} m_e^{m+1}}{2^m \pi^{m/2} \Gamma\left(\frac{m}{2}\right)} \left(\frac{\sigma}{\sigma_c}\right)^{(m+1)/2} \exp\left(-\pi \frac{\sigma_c}{\sigma}\right) \quad (32)$$

The discharging process is taking place until the critical density is reached. Then, it becomes exponentially suppressed. The decay time can be obtained by noticing that

$$\sigma - \sigma_c = \frac{e}{A(r)} \frac{\Delta N}{\Delta \tau} \Delta \tau = e \lambda_e W \Delta \tau \quad (33)$$

Thus, we estimate the discharge time to be

$$\Delta \tau = \left(\frac{2\pi}{m_e} \right)^{m-1} \frac{1}{e^2 \lambda_e} \frac{x-1}{x^{(m+1)/2}} \exp \left(\frac{\pi}{x} \right) \quad (34)$$

where, we introduced the variable $x = \sigma/\sigma_c$. In order to restore the dependence from the minimal length let us recall that the electric charge is a dimensional quantity. Thus, we write $e^2 = 4\pi\alpha_{em} L^{m-3}$ where, $\alpha_{em} = 1/137$ is the fine structure constant in four dimensions, and L is a characteristic length scale of the higher dimensional theory. In our approach there is only one length scale and it is natural to identify $L = \sqrt{\theta}$. Then, the discharge mean time turns out to be (in c.g.s. units)

$$\Delta \tau = \frac{\theta m_e}{\alpha_{em}} \left(\frac{2\pi}{m_e c \sqrt{\theta}} \right)^{m-1} \frac{x-1}{x^{(m+1)/2}} \exp \left(\frac{\pi}{x} \right) \quad (35)$$

where we estimate $m_e c \sqrt{\theta} \approx 0.5 \times 10^{-6}$, which is compatible with expected length scale in TeV quantum gravity. Eq.(35) gives the discharge time assuming that the process takes place in the $m+1$ dimensional bulk spacetime. It is interesting to compare it with corresponding expression for $m=3$ which describes pair creation on the “three-brane” world. Thus we find

$$\frac{\Delta \tau}{\Delta \tau_{m=3}} = \left(\frac{2\pi}{m_e c \sqrt{\theta}} \right)^{m-3} \frac{1}{x^{(m-3)/2}} \quad (36)$$

An upper bound on $\Delta \tau$ can be obtained for $x \approx 1$, i.e. $\sigma \approx \sigma_c$, and $\Delta \tau_{m=3} \leq 1.76 \times 10^{-19} sec$ [28]:

$$\Delta \tau \leq \left(\frac{2\pi}{m_e c \sqrt{\theta}} \right)^{m-3} 1.76 \times 10^{-19} sec \quad (37)$$

As there are no evidence of the presence of a minimal length at the atomic length scale, $m_e c \sqrt{\theta} \ll 1$. Eq.(37) shows that both θ and m contribute to the increase of the mean life by several orders of magnitude, already for m slightly greater than three.

We conclude that: if the decay takes place in the bulk the discharge time increases significantly as a function of m . If, on the contrary, brane universe scenario, where standard model elementary particles are confined on the brane, is realized in nature $\Delta \tau$ should be of the order of $10^{-19} sec$.

Two comments are in order. From (37) we see that Schwinger mechanism is relevant only in the case $m \leq 6$. For higher dimension charge neutralization will take place through Hawking emission, as soon as the BH temperature is higher than 1.22 MeV , and capture of surrounding charged particles produced in the hadronic collision. Secondly, our considerations assume electrons to be free to move in the whole higher dimensional space. Brane models constrain ordinary matter particle to be glued to the brane itself, while only gravity can probe transverse higher dimensions. Even if a D-brane is not explicitly present in our geometry, line element (8) can approximate a D-brane geometry provided the radius of the event horizon is small with respect to the size of the extra dimensions. To take into account models where gauge fields are confined to the brane, Schwinger mechanism must be restricted to take place only inside the brane and the relevant discharge time is given by 10^{-19} s .

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